# Electricity Price Forecasting based on Order Books: a differentiable optimization approach

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# What is differentiable optimization?

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Suppose you have 1M€

$c = 400k€$

$c = 500k€$

$c = 150k€$

$c = 350k€$

$c = 250k€$

# What is differentiable optimization?

Suppose you have 1M€

$c = 400k€$

$\hat{\mathbf{r}} = ?$

$c = 500k€$

$$\widehat{B} = \max_{B_i \in \{0,1\}} \sum B_i \hat{r}_i$$
$$\sum B_i c_i \leqslant 1M€$$

$c = 150k€$

$c = 350k€$

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# What is differentiable optimization?

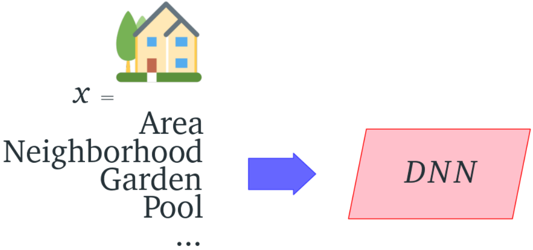
Predict $\hat{r}$

$x =$

Area
Neighborhood
Garden
Pool
...

# What is differentiable optimization?

Predict $\hat{r}$



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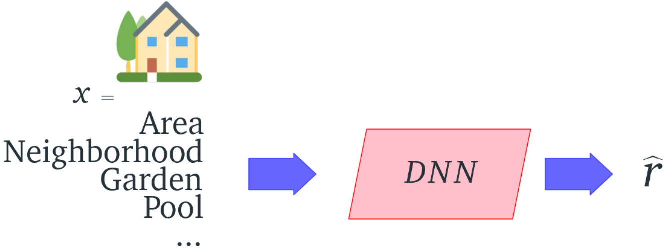

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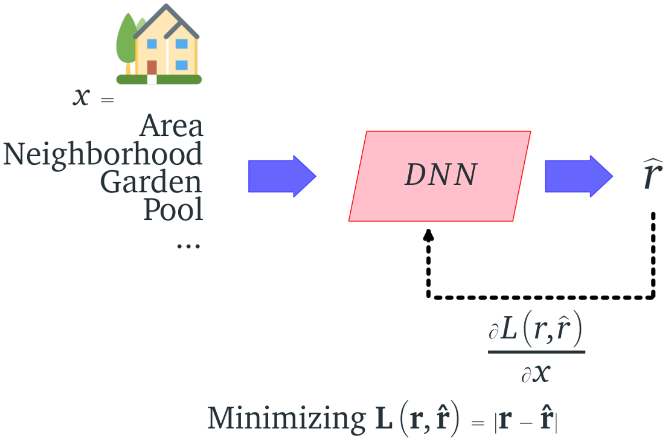

# What is differentiable optimization?

Suppose you have 1M€

$c = 400k€$
$\hat{\mathbf{r}} = \mathbf{425k€}$

$c = 500k€$
$\hat{\mathbf{r}} = \mathbf{575k€}$

$c = 150k€$
$\hat{\mathbf{r}} = \mathbf{225k€}$

$c = 350k€$
$\hat{\mathbf{r}} = \mathbf{400k€}$

$c = 250k€$
$\hat{\mathbf{r}} = \mathbf{300k€}$

$$\widehat{B} = \max_{B_i \in \{0,1\}} \sum B_i \hat{r}_i$$
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$\hat{\mathbf{r}} = 300\text{k€}$

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$$\sum \widehat{B}_i \hat{r}_i = 1.2M€$$

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$\hat{r} = 400k€$
$\mathbf{r} = \mathbf{400k€}$

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$$\widehat{B} = \max_{B_i \in \{0,1\}} \sum B_i \hat{r}_i$$

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$$\sum B_i c_i \leqslant 1M€$$
$$\sum \widehat{B}_i \hat{r}_i = 1.2M€$$
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$$\sum B_i^\star r_i = 1.2M€$$

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$$\sum \widehat{B}_i \hat{r}_i = 1.2M€$$

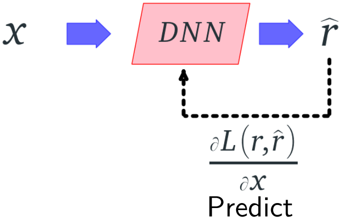
$$\sum \widehat{B}_i r_i = 1.1M€$$

$$\sum B_i^\star r_i = 1.2M€$$

$$REGRET\left(B^\star, \widehat{B}\right) = \sum B_i^\star r_i - \sum \widehat{B}_i r_i = 0.1M€$$

**How do I know that minimizing $L(r, \hat{r}) = |r - \hat{r}|$ will lower the $REGRET$?**

# What is differentiable optimization?



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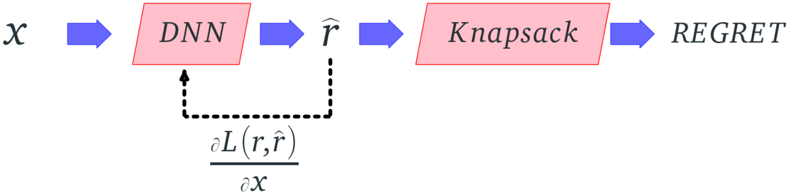


Predict *then* Optimize

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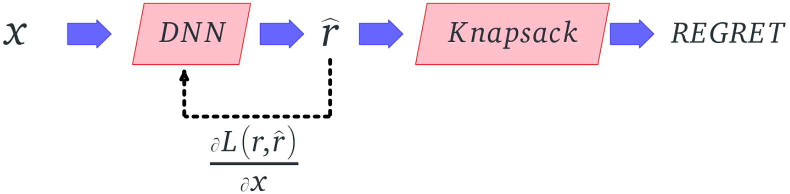


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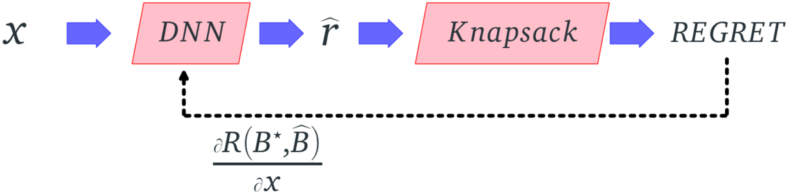

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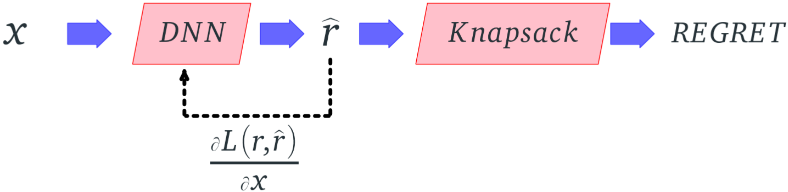


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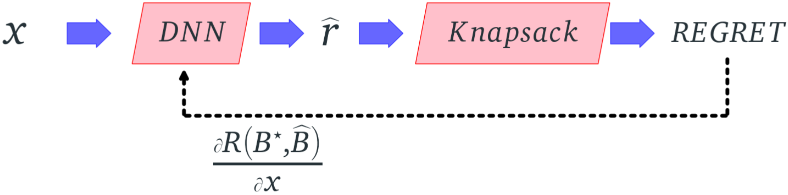


Predict *and* Optimize

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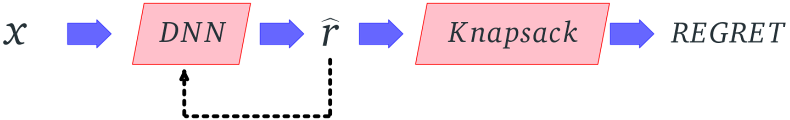
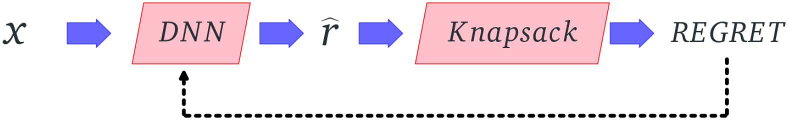



$$\frac{\partial L(r,\hat{r})}{\partial x}$$

Predict *then* Optimize




$$\frac{\partial R(B^\star,\widehat{B})}{\partial x} \propto \frac{\partial R(B^\star,\widehat{B})}{\partial \widehat{B}} \times \frac{\partial \widehat{B}}{\partial \hat{r}} \times \frac{\partial \hat{r}}{\partial x}$$

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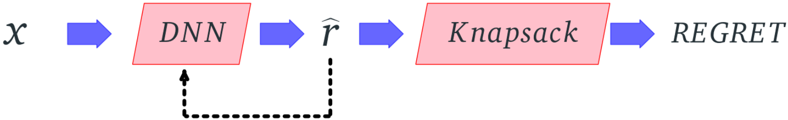


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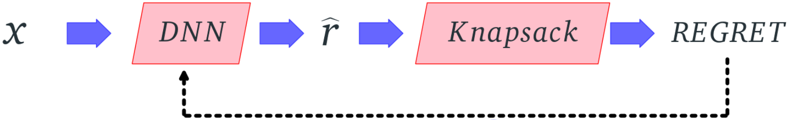


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Differentiable Optimization

# How are Electricity Prices determined?

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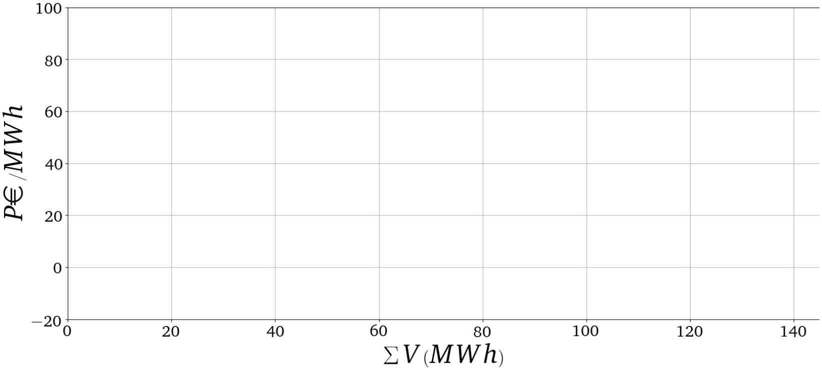

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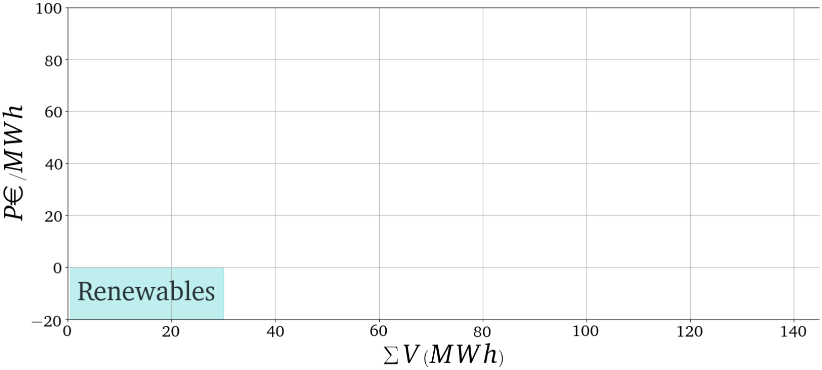

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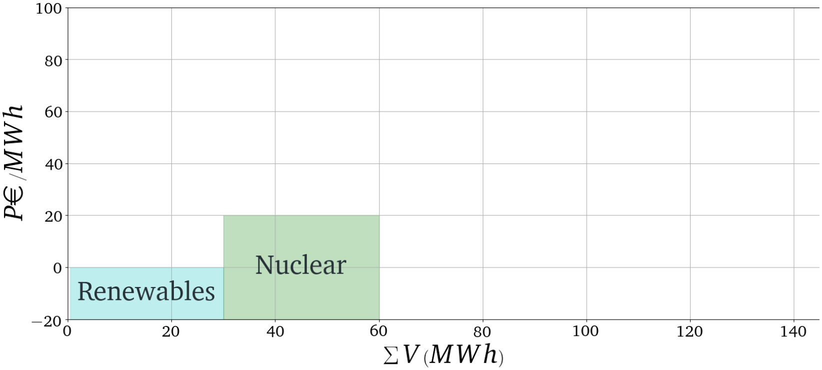

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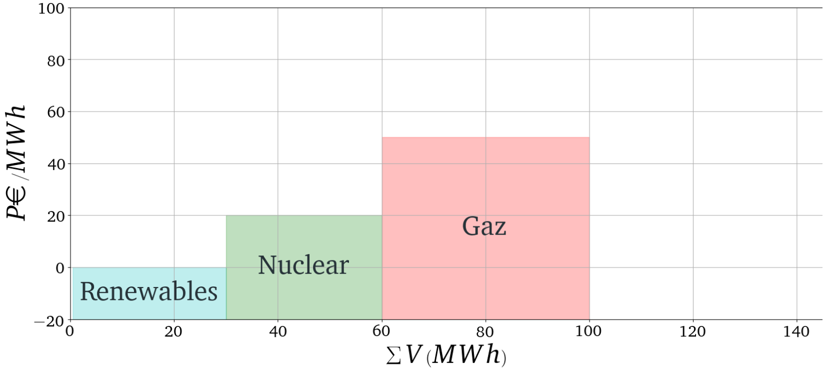

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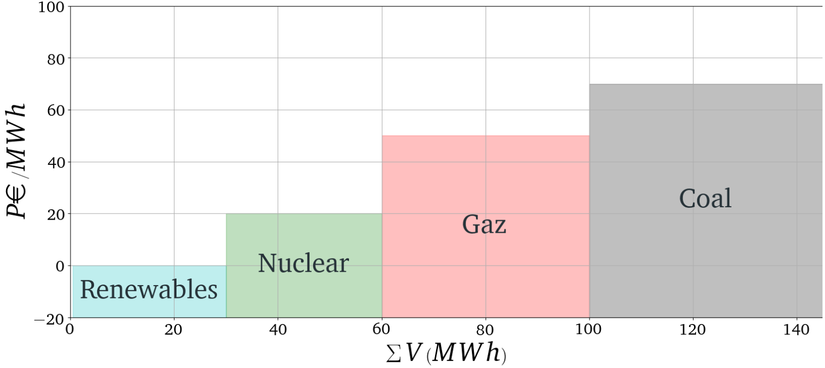

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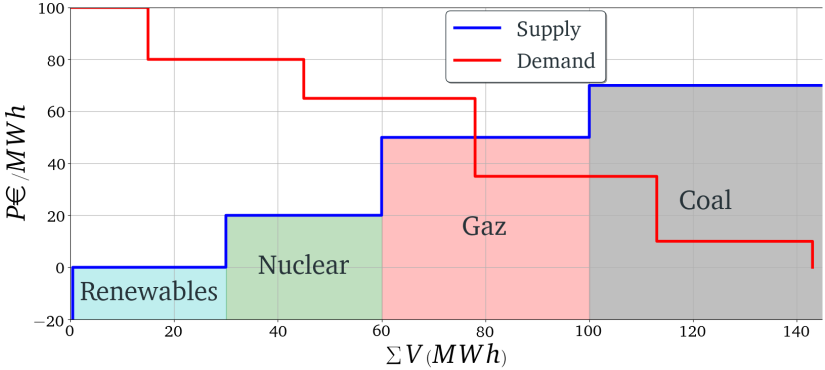

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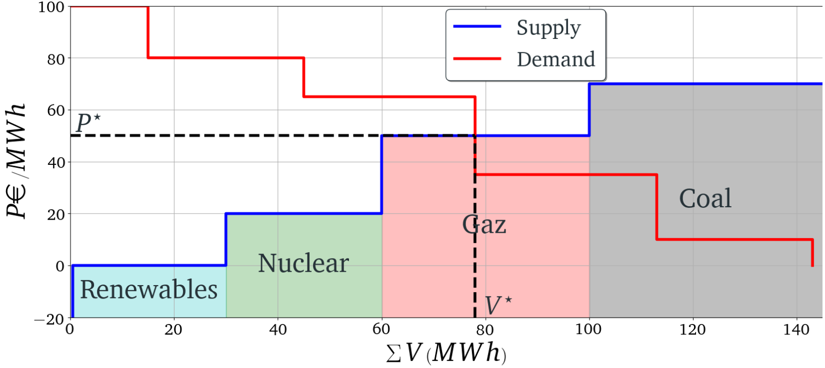
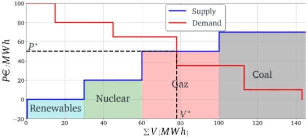


# How are Electricity Prices Determined?

The market uses EUPHEMIA, a Quadratic optimization Problem to set the prices.

Order Books $OB$



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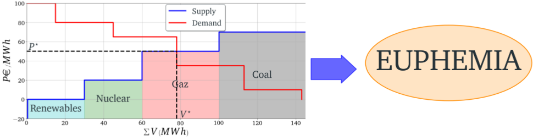


$$\max_{A \in [0,1]} \sum \left( -\frac{1}{2} A_i^2 V_i P_i - A_i V_i P_{o_i} \right)$$

$$\text{u.c.} \quad \sum A_i V_i = 0$$

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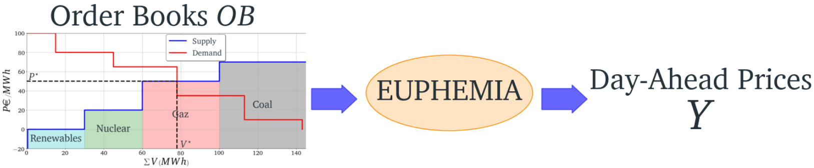


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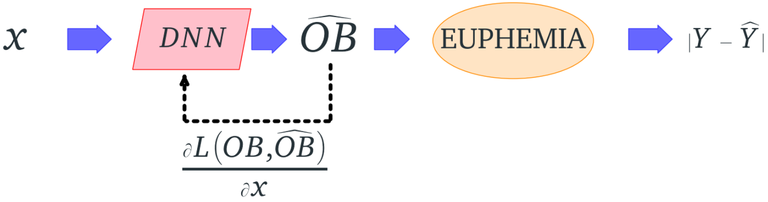
# EUPHEMIA in a price forecasting model



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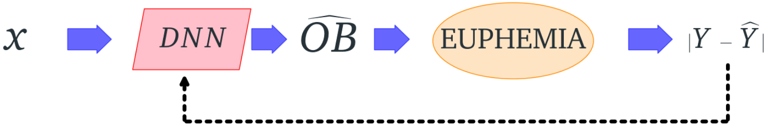


$$\frac{\partial |Y - \widehat{Y}|}{\partial x} = \frac{\partial |Y - \widehat{Y}|}{\partial \widehat{Y}} \times \frac{\partial \widehat{Y}}{\partial \widehat{OB}} \times \frac{\partial \widehat{OB}}{\partial x}$$

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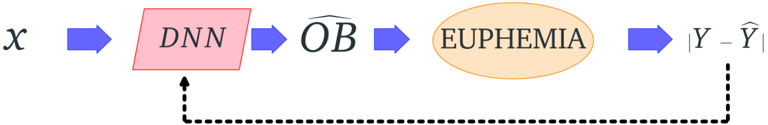


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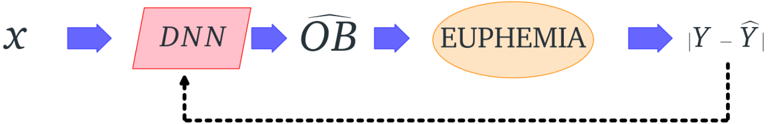


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$$\max_{A \in [0,1]} \sum \left( -\frac{1}{2} A_i^2 V_i P_i - A_i V_i P_{o_i} \right) \leftrightarrow \widehat{Y} = \min_{\lambda \in \mathbb{R}} \mathcal{D}\left(\lambda, \widehat{OB}\right)$$

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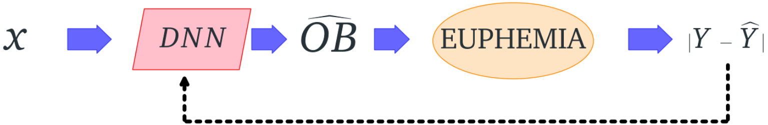


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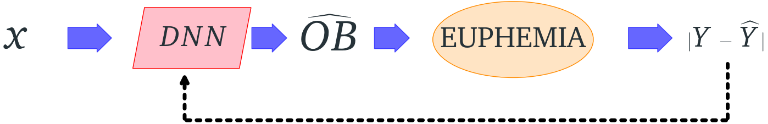


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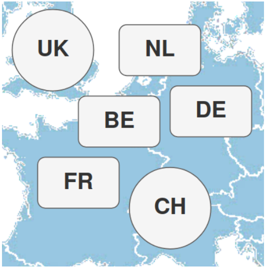
We solve $\mathcal{D}'\left(\lambda, \widehat{OB}\right) = 0$ using a dichotomy search whose gradients can be tracked.

**What is the impact of Differentiable Optimization on the forecasts quality?**

# Experimental results


FR Consumption, Generation & Renewables Forecasts + Past Prices

# Experimental results




- **FR** — Consumption, Generation & Renewables Forecasts + Past Prices
- **UK** — Current Prices available at 11.15am

# Experimental results

| **Country** | Model | | | |
|---|---|---|---|---|
| **BE** | | | | |
| **DE** | | | | |
| **FR** | | | | |
| **NL** | | | | |

# Experimental results

| **Country** | Model | | | |
|---|---|---|---|---|
| **BE** | **DNN** | | | |
| | | | | |
| | | | | |
| **DE** | **DNN** | | | |
| | | | | |
| | | | | |
| **FR** | **DNN** | | | |
| | | | | |
| | | | | |
| **NL** | **DNN** | | | |
| | | | | |
| | | | | |

# Experimental results

| **Country** | Model | | | |
|---|---|---|---|---|
| **BE** | **DNN** | | | |
| | **DO** | | | |
| | | | | |
| **DE** | **DNN** | | | |
| | **DO** | | | |
| | | | | |
| **FR** | **DNN** | | | |
| | **DO** | | | |
| | | | | |
| **NL** | **DNN** | | | |
| | **DO** | | | |
| | | | | |

# Experimental results

| **Country** | Model | | | |
|---|---|---|---|---|
| **BE** | **DNN** | | | |
| | **DO** | | | |
| | **DNN + DO** | | | |
| **DE** | **DNN** | | | |
| | **DO** | | | |
| | **DNN + DO** | | | |
| **FR** | **DNN** | | | |
| | **DO** | | | |
| | **DNN + DO** | | | |
| **NL** | **DNN** | | | |
| | **DO** | | | |
| | **DNN + DO** | | | |

# Experimental results

| **Country** | Model | **MAE** €/MWh | | |
|---|---|---|---|---|
| **BE** | **DNN** | 7.74 | | |
| | **DO** | 7.27 | | |
| | **DNN + DO** | **6.28** | | |
| **DE** | **DNN** | 7.28 | | |
| | **DO** | 9.01 | | |
| | **DNN + DO** | **6.99** | | |
| **FR** | **DNN** | **4.54** | | |
| | **DO** | 6.47 | | |
| | **DNN + DO** | 5.3 | | |
| **NL** | **DNN** | 6.32 | | |
| | **DO** | 6.53 | | |
| | **DNN + DO** | **5.22** | | |

# Experimental results

| **Country** | Model | **MAE** €/MWh | **RMAE** | |
|---|---|---|---|---|
| **BE** | **DNN** | 7.74 | 0.941 | |
| | **DO** | 7.27 | 0.884 | |
| | **DNN + DO** | **6.28** | **0.763** | |
| **DE** | **DNN** | 7.28 | 0.778 | |
| | **DO** | 9.01 | 0.958 | |
| | **DNN + DO** | **6.99** | **0.745** | |
| **FR** | **DNN** | **4.54** | **0.653** | |
| | **DO** | 6.47 | 0.93 | |
| | **DNN + DO** | 5.3 | 0.759 | |
| **NL** | **DNN** | 6.32 | 1.057 | |
| | **DO** | 6.53 | 1.092 | |
| | **DNN + DO** | **5.22** | **0.874** | |

# Experimental results

| **Country** | Model | **MAE** €/MWh | **RMAE** | **SMAPE** % |
|---|---|---|---|---|
| **BE** | **DNN** | 7.74 | 0.941 | 21.27 |
| | **DO** | 7.27 | 0.884 | 19.73 |
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| **FR** | **DNN** | **4.54** | **0.653** | **15.5** |
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# Experimental results

| **Country** | Model | **MAE** €/MWh | **RMAE** | **SMAPE** % |
|---|---|---|---|---|
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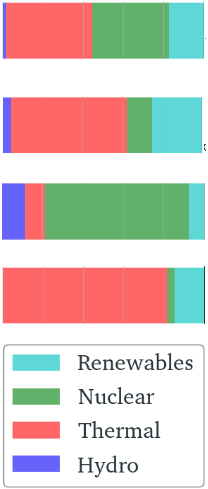
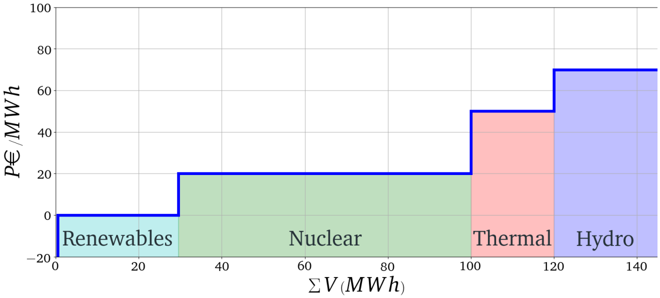
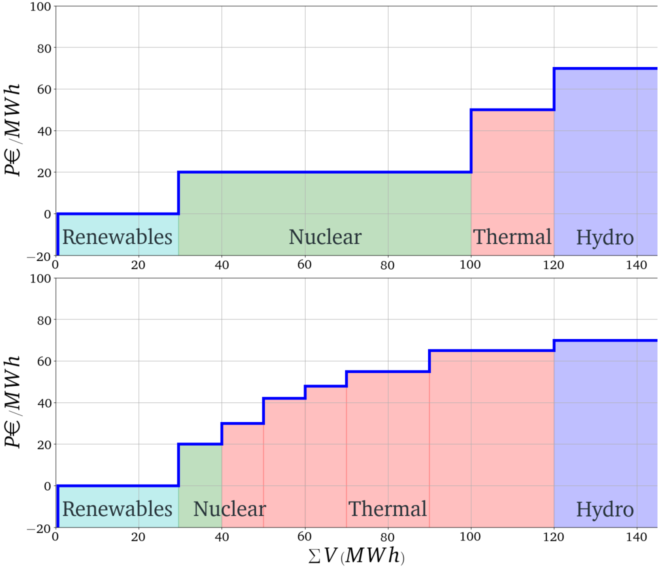
## Energy Mix



# Experimental results



# Experimental results



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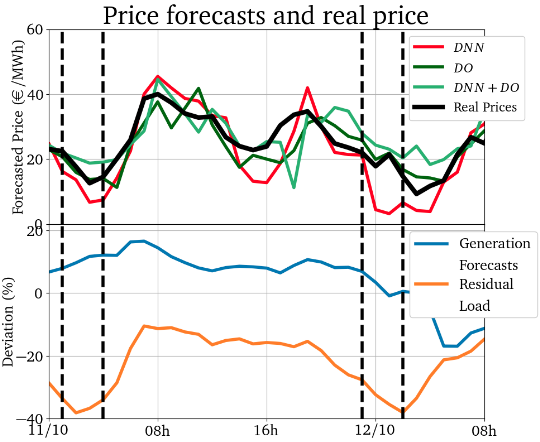

# Thanks for listening!

leonard.tschora@bcmenergy.fr

# Bonus - Formulating the Dual Problem

$$\max_{A \in [0,1]} \sum \left( -\frac{1}{2} A_i^2 V_i P_i - A_i V_i P_{o_i} \right)$$
$$\text{u.c.} \quad \sum A_i V_i = 0$$

$$\mathcal{L}(A, \lambda, M, K) = \sum -\frac{A_i^2 V_i P_i}{2} - A_i V_i Po_i + \lambda A_i V_i - M_i A_i + K_i(Ai - 1)$$

$$\min_{\lambda} \mathcal{D}(\lambda) = \min_{\lambda} \sum \mathcal{D}_i(\lambda) \text{ with } \mathcal{D}_i(\lambda)$$
$$= \begin{cases} (1)\ 0, \text{ if } V_i(Po_i - \lambda) > 0 \\ (2)\ V_i(\lambda - \frac{P_i}{2} - Po_i), \text{ if } V_i(\lambda - P_i - Po_i)) > 0 \\ (3)\ \frac{V_i}{2P_i}(\lambda - Po_i)^2, \text{ if } \lambda \in [Po_i, Po_i + P_i] \end{cases}$$

# Bonus - The Dichotomy Search

Differentiable dichotomy search. H is the Heaviside function.

lb ← -500€/MWh
ub ← 3000€/MWh
found ← **False**
**while** (found = **False**) and $(ub - lb > 2 * 0.01)$ **do**
  $M \leftarrow \frac{ub+lb}{2}$
  $D_M \leftarrow D'(M)$
  found ← $D_M = 0$
  $ub \leftarrow ub - H(D_M) * (ub - M)$
  $lb \leftarrow M - H(D_M) * (M - lb)$
**end while**

# Bonus - The *DNN* + *DO* model